

# Computation of Passive Robustness Bound for Assembly/Disassembly Processes

A. MHALLA<sup>1,2</sup>, S. COLLART DUTILLEUL<sup>3</sup>, M. BENREJEB<sup>1</sup>, E. CRAYE<sup>2</sup>

<sup>1</sup> Unité de Recherche LARA Automatique, Ecole Nationale d'Ingénieurs de Tunis, Université de Tunis el Manar, BP 37, le Belvédère, 1002 Tunis, Tunisie

<sup>2</sup>Laboratoire d'Automatique, Génie Informatique et Signal

Ecole Centrale de Lille, Cité Scientifique BP 48, 59651 Villeneuve d'Ascq, France

<sup>3</sup>Evaluation and Safety of Automated Transport Systems, French institute of science and technology for transport, development and networks, 20 rue Elisée RECLUS BP 70317, F-59666 Villeneuve d'Ascq France

<sup>1,2</sup> anis.mhalla@enim.rnu.tn; <sup>3</sup> simon.collart-dutilleul@ifsttar.fr; <sup>1</sup> mohamed.benrejeb@enit.rnu.tn; <sup>2</sup> etienne.craye@ec-lille.fr

## Abstract

Works presented in this paper deal with the robustness of manufacturing job-shops with time constraints. A computing algorithm of a lower bound of the maximal time disturbances allowed in a given point is provided. To demonstrate the effectiveness and accuracy of this algorithm, two industrial examples are depicted. The possession of this bound allows checking the death of marks without the generation of too many false alarms. From a practical point of view, the death of a mark corresponds to an incorrect treatment of a given product. In food or pharmaceuticals industries, this scenario is not acceptable because it jeopardizes health of humans. When a disturbance is lower than the bound value, the production remains correct and no quality alarm is generated.

## Keywords

*P-time Petri Net; Manufacturing System; Passive Robustness; Sojourn Time Constraints; Time Disturbance*

## Introduction

This paper deals with control over manufacturing workshops with time constraints. The considered systems have a robustness property which allows maintaining products quality when there are time disturbances (Jerbi et al, 2009). The robustness is defined as the capability of the system to preserve the specifications when confront some expected or unexpected variations. Thus, the robustness characterizes the capacity to deal with disturbances. The robustness is interpreted into different specializations. The passive robustness is based upon variations included in validity time intervals. There is no control loop modification to preserve the required specifications. On the other hand, active robustness uses observed time disturbances to modify the control loop in order to satisfy these specifications. During the

past decade, several approaches have dealt with manufacturing workshops with time constraints (Jerbi et al, 2004), (Atto et al, 2008). For example, in Jerbi (2004), a computation of the local passive robustness of a given path is built up. Moreover, an algorithm computing a lower bound of the maximal time disturbances allowed on a given point was provided. Since, the result of the algorithm is a lower bound.

The work presented in this paper focuses on the robustness of workshops with/without assembling tasks, regarding time disturbances. More precisely, we aim at computing the value of passive robustness bounds for some systems characterized with synchronization and delay phenomena. Among the broad variety of applications concerned, one can cite transportation systems and production systems. The contribution of this paper is the application of a computing algorithm with a lower bound of the maximal time disturbances to workshops with/without assembling tasks.

The remainder of this paper is organized as follows. Section 1 begins with the definition of a P-time Petri net model and its functional decomposition. Section 2 gives some basic definitions concerning robustness of Discrete Event Systems (DES). Then, the local robustness of a given path in the workshop is analytically built up. Moreover, an algorithm computing a lower bound of the maximal time disturbances allowed on a given point is provided. Finally, in order to show the effectiveness of this algorithm, an application of the proposed algorithm to two manufacturing workshops is outlined and results are discussed.

## Modelling tools of DES Integrating Time Constraints

### P-time Petri Net

**Definition 1 (Khansa et al, 1996):** The formal definition of a P-time Petri net is given by a pair  $\langle R; I \rangle$  where:

$R$  is a marked Petri net,

$IS: P \rightarrow Q^+ \times (Q^+ \cup \{+\infty\})$

$p_i \rightarrow IS_i = [a_i, b_i]$  with  $0 \leq a_i \leq b_i$ .

$IS_i$  defines the static interval of holding time of a mark in the place  $p_i$  belonging to the set of places  $P$  ( $Q^+$  is the set of positive rational numbers). A mark in the place  $p_i$  is taken into account in transition validation when it has stayed in  $p_i$  at least a duration  $a_i$  and no longer than  $b_i$ . After the duration  $b_i$  the token will be dead.

### Notations

- $p_i^o$  (respectively  $^o p_i$ ): the output transitions of the place  $p_i$  (the input transitions of the place  $p_i$ ),
- $q_{ie}$ : the expected sojourn time of the token in the place  $p_i$ ,
- $q_i$ : the effective sojourn time of the token in the place  $p_i$ ,
- $St_e(n)$ : the  $n$ nd expected firing instant of the transition  $t$ ,
- $St(n)$ : the  $n$ nd effective firing instant of the transition  $t$ ,
- $T_c$ : the set of controllable transitions,
- $T_s$ : the set of synchronization transitions.

### Functional Decomposition

As the sojourn times in places have not the same functional signification when they are included in the sequential process of a product or when they are associated to a free resource, a decomposition of the Petri net model into four sets is made using (Long, 1993), where:

- $R_U$  is the set of places representing the used machines,
- $R_N$  corresponds to the set of places representing the free machines
- $Trans_c$  is the set of places representing the loaded transport resources,
- $Trans_{NC}$  is the set of places representing the unloaded transport resources (or the interconnected buffers),
- $Trans_c$  is the set of places representing the loaded transport resources.

### Passive Robustness

#### Basic Definition

**Definition 2 (Jerbi et al 2004):** Let us consider a discrete event system and  $G$  the associated Petri net model. Let

us call  $B(G)$  the behaviour of  $G$  corresponding to the trajectory that states successively reached. Let  $C(B(G))$  be the schedule of conditions established on the system behaviour  $B(G)$ .  $C(B(G))$  is materialized by a series of constraints which must be checked by  $B(G)$ . A non respect of  $B(G)$  corresponds to a violation of  $C(B(G))$ . It is said that a subset  $SG$  of  $G$  is robust to a disturbance  $\delta$  if and only if  $\forall n \in SG$ , and that the occurrence of the disturbance  $\delta$  at the node  $n$  does not involve a violation of  $C(B(G))$ .

**Definition 3 (Collart Dutilleul, 2007):** The passive robustness corresponds if no change in control is necessary so that the properties specified by the schedule of conditions are preserved in the presence of disturbance.

**Definition 4 (Collart Dutilleul, 2007):** A mono-synchronized subpath  $L_p$  is a path containing one and only one synchronization which is its last node.

**Definition 5:** An elementary mono-synchronized subpath is a mono-synchronized subpath beginning with a place  $p$  such as  $^o p$  is a synchronization transition.

**Definition 6:** It is said that a path  $L_p$  has a local passive robustness on  $[\delta_{\min}, \delta_{\max}]$  if the occurrence of a disturbance  $\delta \in [\delta_{\min}, \delta_{\max}]$  at any place  $p \in L_p$  does not involve a token death at synchronization transitions of  $L_p$ .

Figure 1, shows an elementary mono-synchronized subpath  $L_p = (p_{112}, t_{112}, p_{122}, t_{12})$  with different notations.

**Definition 7 (Jerbi et al, 2009):** The time passive rejection capacity interval of a path  $L_p$  is  $RC(L_p) = [Ca(L_p), Cr(L_p)]$  where:

$$Ca(L_p) = \sum_{p_i \in L_p \cap (R_N \cup Trans_{NC})} (q_{ie} - b_i), \quad (1)$$

$$Cr(L_p) = \sum_{p_i \in L_p \cap (R_N \cup Trans_{NC})} (q_{ie} - a_i). \quad (2)$$

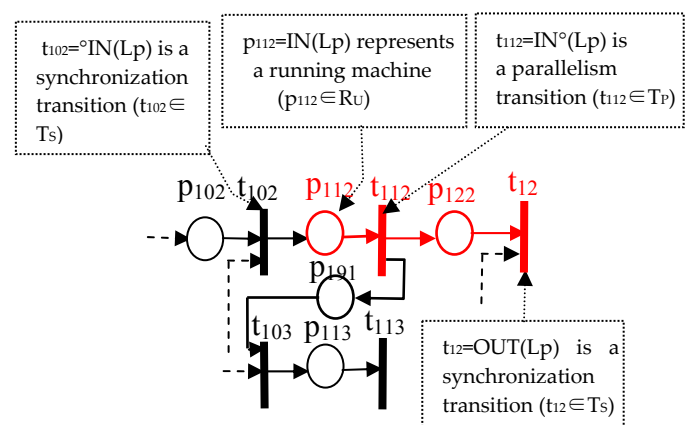


FIG. 1 AN ELEMENTARY MONO-SYNCHRONIZED SUBPATH WITH DIFFERENT NOTATIONS

### Passive Robustness Computation

To compute local passive robustness interval, the concept of compensable and transmissible margin is introduced.

The compensable margin and the transmissible margin on the mono-synchronized subpath  $Lp_k$  are denoted as  $\Delta rc_k$  and  $\Delta rt_k$  respectively.

The local passive robustness delay  $\Delta r_{Lp_k}$  can be calculated using formulas (3), (4) and (5) (Collart Dutilleul et al, 2007).

$$\Delta r_{Lp_k} = \Delta rc_k + \Delta rt_k \quad (3)$$

With:

$$\Delta rc_k = \sum_{p_i \in Lp_k \cap (R_N \cup Trans_{NC})} (q_{ie} - a_i) \quad (4)$$

$$\Delta rt_k = \min_{\substack{p_i^o = OUT(Lp_k) \\ p_i \notin Lp_k}} (b_i - q_{ie}) \quad (5)$$

#### 1) Computation Algorithm of the Lower Value of Passive Robustness for a Delay

In order to avoid the violation of schedule conditions, a recursive algorithm allowing computing a lower bound of the maximal time disturbances allowed in a given point of manufacturing workshops is presented.

##### a) Algorithm (Jerbi et al, 2004)

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 $\varphi = \{Lp_j / (n^o = IN(Lp_j)) \wedge (Lp_j \in C_{ms}) \wedge (Lp_j \in G)\}$ 

Marge  $\leftarrow \min_j [\Delta rc_j + F(G \setminus Lp_j, OUT(Lp_j)^o, \min(b_i - q_{ie}))]$ 

 $F(G^*, p^*, \Delta rt)$ 
{
   $\varphi^* = \{Lp_j / (p^* \in Lp_j) \wedge (Lp_j \in C_{se}) \wedge (Lp_j \in G^*)\}$ 
  If  $(\varphi^* = \Phi \text{ ou } \Delta rt = 0)$  alors  $(F \leftarrow \Delta rt)$ 
  Else
  {
     $F \leftarrow \min_j [\min(\Delta rt, (\Delta rc_j + F(G^* \setminus Lp_j, OUT(Lp_j)^o, \min(b_i - q_{ie}))))]$ 
  }
}

```

#### 2) Description of the Algorithm

The algorithm behaves in such a way:

- Select the node where we want to calculate the passive robustness margin,
- Build the set of mono-synchronized subpaths ( $\varphi$ ) defined as follows:

$$\varphi = \{Lp_j / (n^o = IN(Lp_j)) \wedge (Lp_j \in C_{ms}) \wedge (Lp_j \in G)\}$$

- Calculate the passive robustness margin associated to the set of subpaths  $\varphi$ ,

- Remove the elementary mono-synchronized subpath for the construction of the whole  $\varphi^*$  defined as follows:

$$\varphi^* = \{Lp_j / (p^* \in Lp_j) \wedge (Lp_j \in C_{se}) \wedge (Lp_j \in G^*)\}$$

- Compute for each mono-synchronized subpath of  $\varphi^*$ , the passive robustness margin  $F$  defined as follows:

$$F \leftarrow \min_j \{ \min(\Delta rt, (\Delta rc_j + F(G^* \setminus Lp_j, OUT(Lp_j)^o, \min(b_i - q_{ie})))) \}$$

- Stop the algorithm if the following condition is satisfied:

$$(\varphi^* = \Phi \text{ ou } \Delta rt = 0) \text{ alors } (F \leftarrow \Delta rt)$$

### Illustrative Examples

#### Milk Production Unit (Job Shop with Assembling Tasks)

##### 1) Presentation of the Workshop

Figure 2, shows a milk manufacturing unit in repetitive functioning mode, composed by six machines ( $M_1, M_2, M_3, M_4, M_5, M_6$ ) and seven conveyors ( $T_1, T_2, T_3, T_4, T_5, T_6, T_7$ ), where:

- $M_1$  is a bottle filling machine,
- $M_2$  is a milk bottle capper,
- $M_3$  is a time/date stamp,
- $M_4$  is an hydromat,
- $M_5$  is a labelling machine,
- $M_6$  is a packaging machine.

To manufacture product (bottle of 1000 ml), empty bottles (group of 6) are placed on the conveyor  $T_1$  to supply the bottle filling machine  $M_1$ . Once this operation is completed, the filled bottles are transported towards the capping machine  $M_2$  by the conveyor  $T_2$ . After capping, the bottles arrive directory on  $T_3$ , then this conveyor carries the bottles to the machine  $M_3$  (time/date stamp) to print the manufacturing date and end date of consumption.

The bottles leave the machine  $M_3$  on a conveyor  $T_4$  towards the machine  $M_4$  (Hydromat). The hydromat ensures the sterilization of milk which is heated to sterilization temperature  $140^{\circ}\text{--}150^{\circ}\text{C}$ . After sterilization, cooling takes place using a cold water tank and atmospheric air. As a result, the milk does not require refrigeration and has a relatively longer shelf life (3 to 6 months at ambient temperature) with an excellent safeguarding of the vitamins and original qualities. Once this task is completed, the bottles are transferred to the labelling machine  $M_5$  via the conveyor  $T_5$ . Next; the bottles arrive to the packaging machine  $M_6$ , where they will be wrapped by welding in a group of 6. Lastly, the finished product is deposited on the conveyor  $T_7$  towards the stock of finished products SA.

## 2) Modeling of Milk Production Unit

P-time Petri nets are convenient tools to model manufacturing systems whose activities times are included between a minimum and a maximum value. Figure 3 shows a P-time Petri net ( $G$ ) modeling a milk production workshop and its functional decomposition. The obtained  $G$  is used to study the robustness of the considered manufacturing workshop (Long, 1993).

## 3) Computation of the Lower Value of Passive Robustness in Milk Manufacturing Workshop

In Table I, we can quote the following examples of the compensable margin and the transmissible margin associated to some mono-synchronized

subpaths  $Lp_k$  of the P-time Petri net model (Figure 3).

Let us take the P-time Petri net as example, Figure 3 associated to milk manufacturing unit.

The application of the algorithm at the node  $t_9$ , in Figure 4, allows computing a lower value of passive robustness margin equal to 10 (Margin= 10).

In fact, according to the recursive algorithm, the set of mono-synchronized subpaths ( $\phi$ ) is defined as follows (Figure 4):

$$\phi = \{Lp_i / (n^{\circ} = \text{IN}(Lp_i)) \wedge (Lp_i \in C_{ms}) \wedge (Lp_i \in G)\}$$

$$\phi = \{Lp_0=(p_9, t_9, p_{18}, t_8); Lp_1=(p_9, t_9, p_{101}, t_{101}); Lp_2=(p_9, t_9, p_{102}, t_{102}); Lp_3=(p_9, t_9, p_{103}, t_{103}); Lp_4=(p_9, t_9, p_{104}, t_{104}); Lp_5=(p_9, t_9, p_{105}, t_{105}); Lp_6=(p_9, t_9, p_{106}, t_{106})\}.$$

TABLE I Compensable margin and transmissible margin associated to some mono-synchronized subpaths of  $G$

| Path   | $\Delta r_{t_k}$ | $\Delta r_{c_k}$ |
|--|------------------|------------------|
| $Lp_0=(p_9, t_9, p_{18}, t_8)$                 | 6                | 689              |
| $Lp_1=(p_9, t_9, p_{101}, t_{101})$            | $+\infty$        | 0                |
| $Lp_2=(p_9, t_9, p_{102}, t_{102})$            | $+\infty$        | 0                |
| $Lp_3=(p_9, t_9, p_{103}, t_{103})$            | $+\infty$        | 0                |
| $Lp_4=(p_9, t_9, p_{104}, t_{104})$            | $+\infty$        | 0                |
| $Lp_5=(p_9, t_9, p_{105}, t_{105})$            | $+\infty$        | 0                |
| $Lp_6=(p_9, t_9, p_{106}, t_{106})$            | $+\infty$        | 0                |
| $Lp_7=(p_{111}, t_{111}, p_{191}, t_{102})$    | 56               | 5                |
| $Lp_8=(p_{112}, t_{112}, p_{192}, t_{103})$    | 41               | 3                |
| $Lp_9=(p_{113}, t_{113}, p_{193}, t_{104})$    | 28               | 3                |
| $Lp_{10}=(p_{114}, t_{114}, p_{194}, t_{105})$ | 19               | 2                |
| $Lp_{11}=(p_{115}, t_{115}, p_{195}, t_{106})$ | 12               | 4                |
| $Lp_{12}=(p_{116}, t_{116}, p_{196}, t_{101})$ | 71               | 36               |

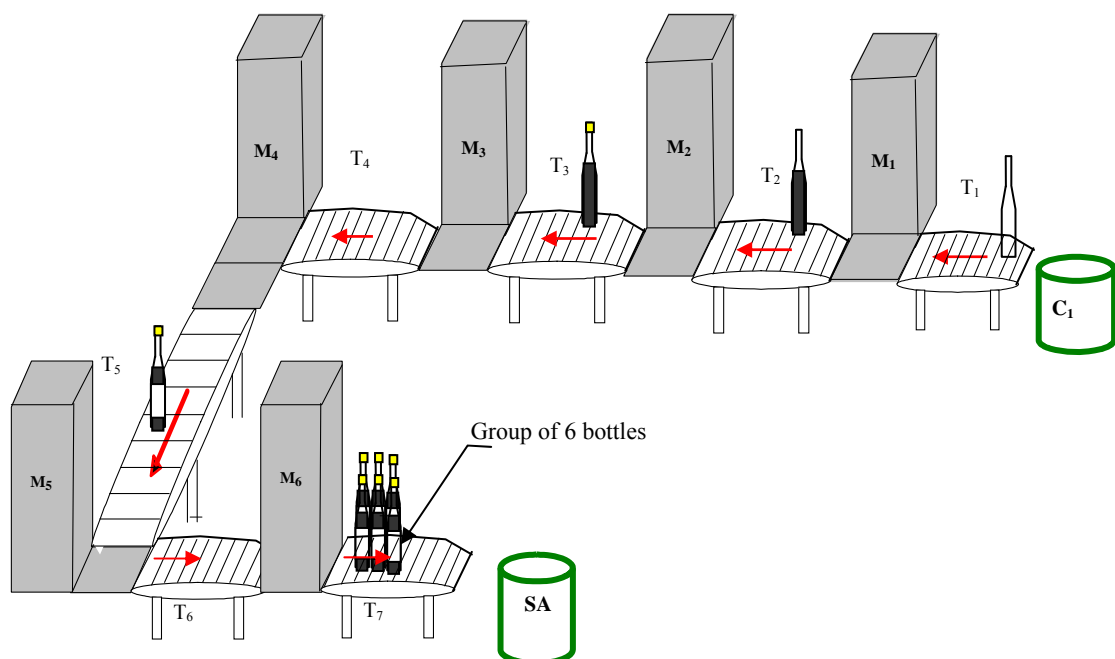


FIG.2 MILK MANUFACTURING UNIT

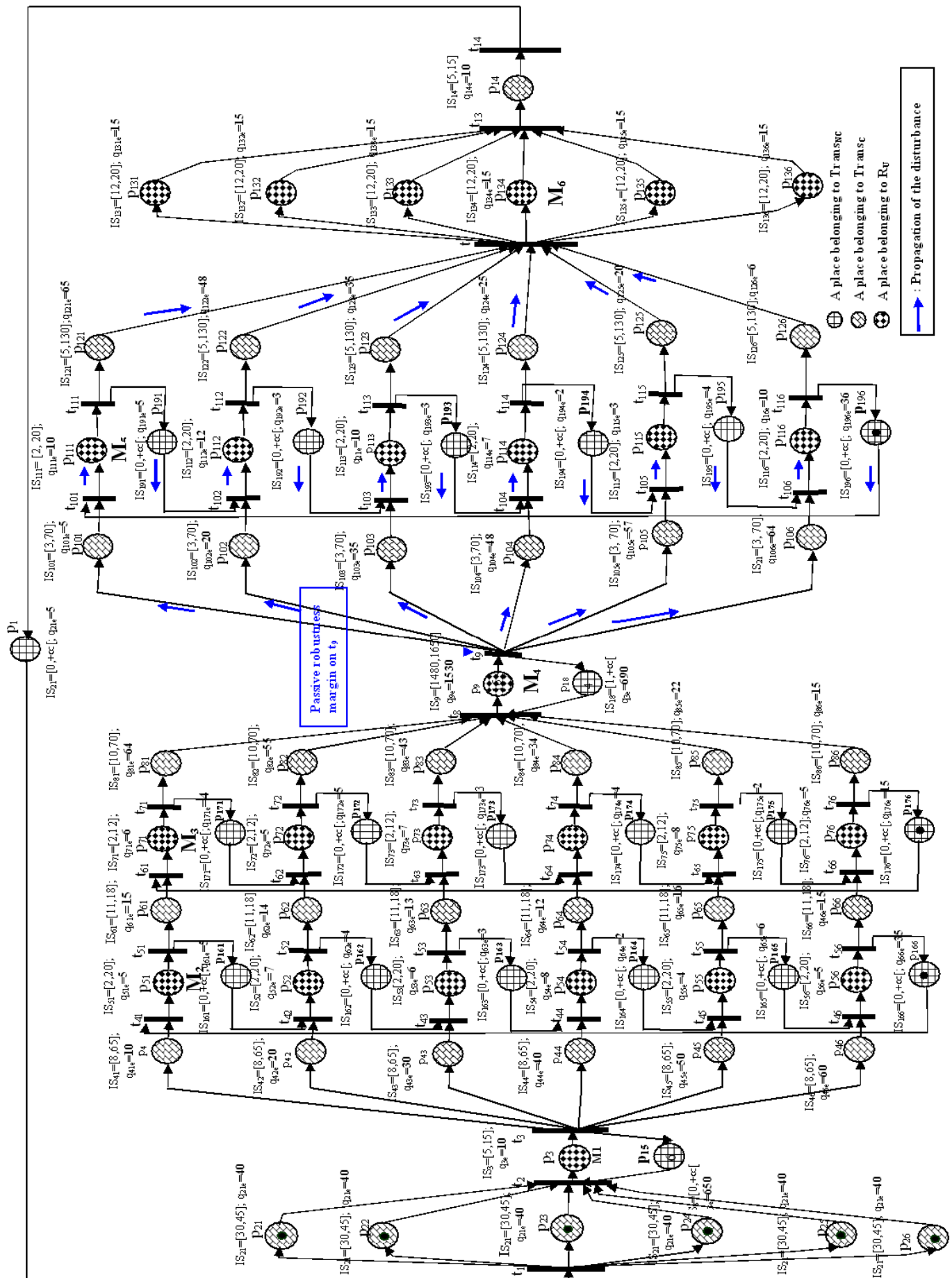


FIG. 3 A MILK MANUFACTURING UNIT MODELLED BY A P-TIME PETRI NET

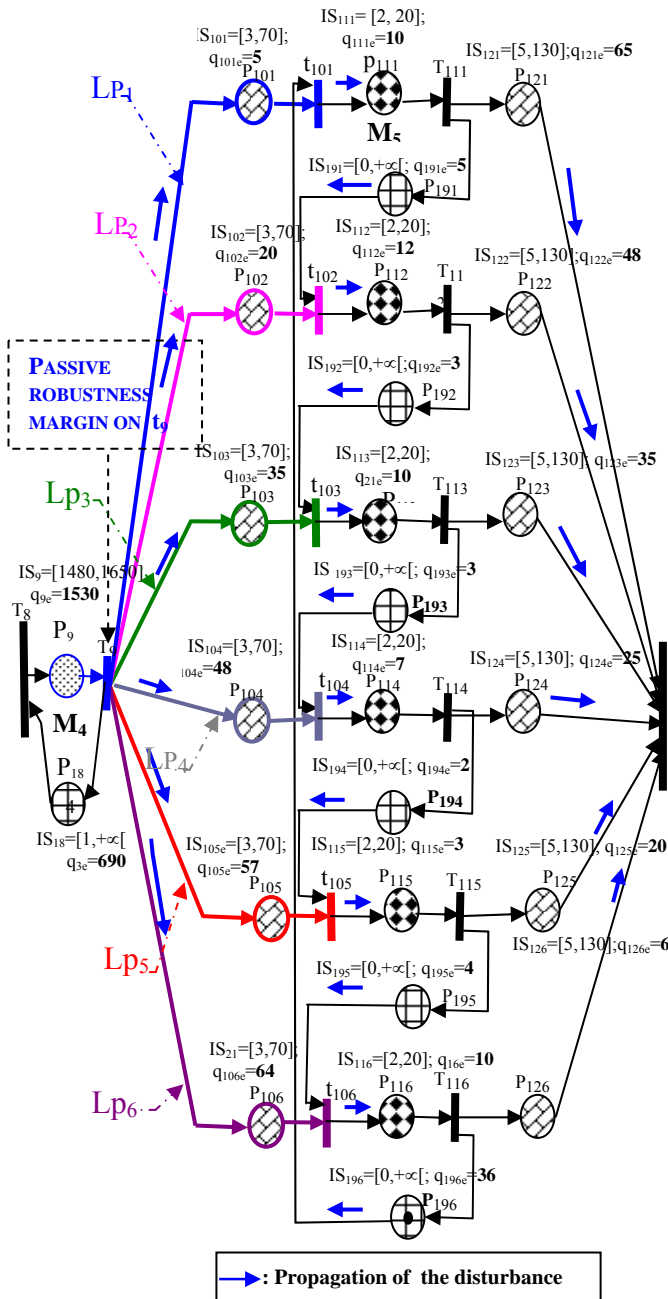


FIG. 4 EXAMPLE OF DISTURBANCE PROPAGATION: CASE OF PASSIVE ROBUSTNESS

The full set of margins associated to each mono-synchronized subpath is summarized in table II. The lower bound of passive robustness margin is equal to 10 ( $F \leftarrow \min [\text{Margin 1, Margin 2, Margin 3, Margin 4, Margin 5, Margin 6}]$ ).

Let us illustrate the local margin associated to the path  $Lp_1$ . This margin equals 23 time units;

- On the path  $Lp_1$ , the disturbance change passively the firing instant of the transition  $t_9$  and also the sojourn time in the place  $p_9$ :  $St_9(n) = St_9(n) + 23$  and  $q_9 = q_9 + 23 = 1553$ .

TABLE II Passive margin associated to mono-synchronized subpaths

| Margin  | Expression  | Value       |
|---------|---|-------------|
| Margin1 | Margin 1 $\leftarrow \min [\Delta rc_1 + F(G \setminus Lp_1, \text{OUT}(Lp_1)^\circ, \min (b_{196} - q_{196e}))]$<br>$p_{196}^\circ = \text{OUT}(Lp_1)$<br>Margin 1 $\leftarrow \min [0 + F(G \setminus Lp_1, p_{111}, +\infty)]$ | Margin1=23  |
| Margin2 | Margin 2 $\leftarrow \min [\Delta rc_2 + F(G \setminus Lp_2, \text{OUT}(Lp_2)^\circ, \min (b_{191} - q_{191e}))]$<br>$p_{191}^\circ = \text{OUT}(Lp_2)$<br>Margin 2 $\leftarrow \min [0 + F(G \setminus Lp_2, p_{112}, +\infty)]$ | Margin2=18  |
| Margin3 | Margin 3 $\leftarrow \min [\Delta rc_3 + F(G \setminus Lp_3, \text{OUT}(Lp_3)^\circ, \min (b_{192} - q_{192e}))]$<br>$p_{192}^\circ = \text{OUT}(Lp_3)$<br>Margin 3 $\leftarrow \min [0 + F(G \setminus Lp_3, p_{113}, +\infty)]$ | Margin3=15  |
| Margin4 | Margin 4 $\leftarrow \min [\Delta rc_4 + F(G \setminus Lp_4, \text{OUT}(Lp_4)^\circ, \min (b_{193} - q_{193e}))]$<br>$p_{193}^\circ = \text{OUT}(Lp_4)$<br>Margin 4 $\leftarrow \min [0 + F(G \setminus Lp_4, p_{114}, +\infty)]$ | Margin4=12  |
| Margin5 | Margin 5 $\leftarrow \min [\Delta rc_5 + F(G \setminus Lp_5, \text{OUT}(Lp_5)^\circ, \min (b_{194} - q_{194e}))]$<br>$p_{194}^\circ = \text{OUT}(Lp_5)$<br>Margin 5 $\leftarrow \min [0 + F(G \setminus Lp_5, p_{115}, +\infty)]$ | Margin5=10  |
| Margin6 | Margin 6 $\leftarrow \min [\Delta rc_6 + F(G \setminus Lp_6, \text{OUT}(Lp_6)^\circ, \min (b_{195} - q_{195e}))]$<br>$p_{195}^\circ = \text{OUT}(Lp_6)$<br>Margin 6 $\leftarrow \min [0 + F(G \setminus Lp_6, p_{116}, +\infty)]$ | Margin6=106 |

- After the crossing of the transition  $t_{101}$ , the disturbance is transmitted to the two paths  $Lp_7 = (p_{111}, t_{111}, p_{191}, t_{102})$  and  $Lp_{13} = (p_{111}, t_{111}, p_{121}, t_{12})$  through the starting place  $p_{111}$ .

- On the path  $Lp_7$ , the disturbance is partially rejected in  $p_{191}$  ( $Cr(Lp_7) = 5$ ) and the mark is available in  $p_{191}$  with a delay time equals to 18. This delay is accepted in  $p_{102}$  since the available control margin for a delay accepted is equals to 50 ( $IS_{102} = [3, 70]$ ,  $q_{102e} = 20$ ). The firing instant of the transition  $t_{101}$  is  $St_{101}(n) = St_{101e}(n) + 18$  and the residue ( $\delta' = 18$ ) is transmitted to the two paths  $Lp_8 = (p_{112}, t_{112}, p_{192}, t_{103})$  and  $Lp_{14} = (p_{112}, t_{112}, p_{122}, t_{12})$  through the place  $p_{112}$ .

- On the path  $Lp_8$ , the residue of disturbance is partially rejected ( $Cr(Lp_8) = 3$ ), and this delay is acceptable on  $p_{103}$  since the passive transmissible margin on the mono-synchronized subpath  $Lp_3$  is 35 ( $IS_{103} = [3, 70]$ ,  $q_{103e} = 35$ ).

- The residue  $\delta'' = 15$  is propagated towards the three paths  $Lp_9 = (p_{113}, t_{113}, p_{193}, t_{104})$ ,  $Lp_{10} = (p_{114}, t_{114}, p_{194}, t_{105})$  and  $Lp_{11} = (p_{115}, t_{115}, p_{195}, t_{106})$ . It is easily to check that this residue is partially rejected by these three paths since  $Cr(Lp_9) = 3$ ,  $Cr(Lp_{10}) = 2$ , and  $Cr(Lp_{11}) = 4$ .

- The token is available in  $p_{195}$  with a delay time equals to 6. There is no death of mark in  $p_{106}$  since



we can accept a maximum delay time equal to 6 ( $IS_{106}=[3, 70]$ ,  $q_{106e}=64$ ). Therefore, the occurrence of the disturbance  $\delta$  at the node  $t_9$  does not involve a violation of  $C(B(G))$ .

### Multi-product Job-shops without Assembling Tasks

Figure 5 shows a P-time Petri net (G) modeling a system composed of two sequential processes  $GO_1$  and  $GO_2$  with two shared machines ( $M_1, M_2$ ).

The full set compensable margin and the transmissible margin associated to some mono-synchronized subpaths  $Lp$ , in Figure 5, are summarized in Table III.

The application of the algorithm at the node  $t_5$ , in Figure 5, allows computing a passive robustness margin (lower bound) equal to 6 (Margin=6). It is easily to check that the occurrence of the disturbance at the node  $t_5$  does not involve a violation of  $C(B(G))$ .

- The residue  $\delta''=15$  is propagated towards the three paths  $Lp_9=(p_{113}, t_{113}, p_{193}, t_{104})$ ,  $Lp_{10}=(p_{114}, t_{114}, p_{194}, t_{105})$  and  $Lp_{11}=(p_{115}, t_{115}, p_{195}, t_{106})$ . It is easily to check that this residue is partially rejected by these tree paths since  $Cr(Lp_9)=3$ ,  $Cr(Lp_{10})=2$ , and  $Cr(Lp_{11})=4$ . The token is available in  $p_{195}$  with a delay time equal to 6. There is no death of mark in  $p_{106}$  since we can accept a maximum delay time equal to 6 ( $IS_{106}=[3, 70]$ ,  $q_{106e}=64$ ). Therefore, the occurrence of the disturbance  $\delta$  at the node  $t_9$  does not involve a violation of  $C(B(G))$ .

**Margin**  $\Leftarrow F(G \setminus Lp', p_2, +\infty)$

$\Phi^* = \{Lp_3, Lp_4\}$  et ( $L_1 = Lp' \cup Lp_3$ ;  $L_2 = Lp' \cup Lp_4$ )

$$F \Leftarrow \min \begin{cases} \min[+\infty, F(G \setminus Lp' \setminus Lp_3, p_4, +\infty)] \\ \min[+\infty, 5+F(G \setminus Lp' \setminus Lp_4, p_{13}, 1)] \end{cases}$$

**Step 1:** Computation of  $F(G \setminus Lp' \setminus Lp_3, p_4, +\infty)$

$\Phi^* = \{Lp_6\}$

$F \Leftarrow \min[+\infty, 5+F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6, p_{11}, 3)]$

**Step 1.1:** Computation of  $F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6, p_{11}, 3)$

$$F \Leftarrow \min \begin{cases} \Phi^* = \{Lp_7, Lp_8\} \\ \min[3, 8+F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_7, p_4, 5)] = 3 \\ \min[3, F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_8, p_{13}, +\infty)] \end{cases}$$

**Step 1.1.1:** Computation of  $F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_8, p_{13}, +\infty)$

$$F \Leftarrow \min \begin{cases} \Phi^* = \{Lp_1, Lp_2\} \\ \min[+\infty, 18+F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_8 \setminus Lp_1, p_{11}, +\infty)] \\ \min[+\infty, 7+F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_8 \setminus Lp_2, p_2, 9)] \end{cases}$$

**Step 1.1.1.1:** Computation of  $F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_8 \setminus Lp_1, p_{11}, +\infty)$

$\Phi^* = \Phi$  then  $F \Leftarrow +\infty$

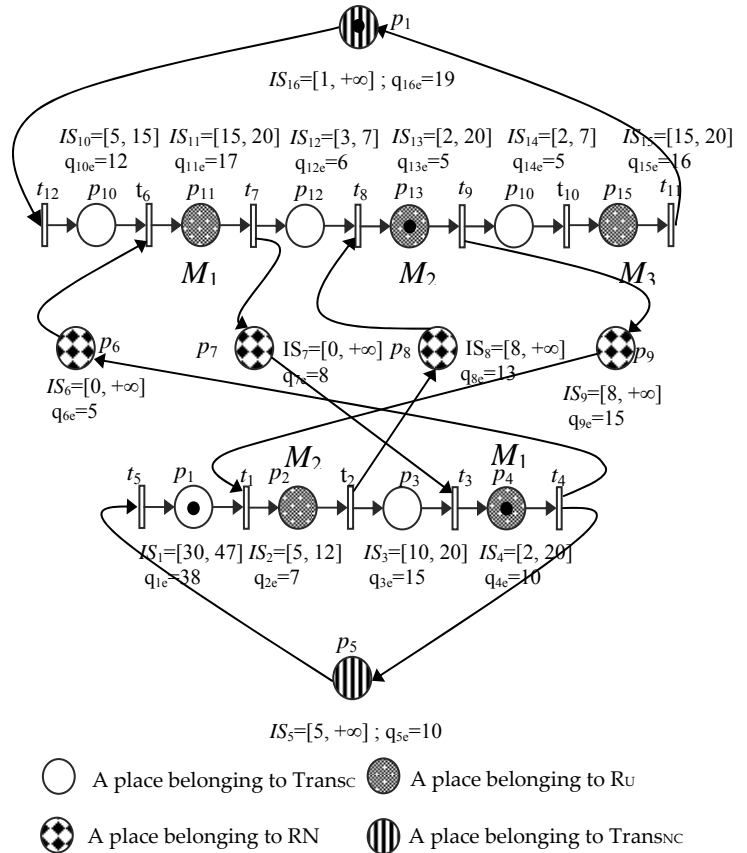


FIG. 5 AN HILLION LIKE MODEL

TABLE III PASSIVE MARGIN ASSOCIATED TO MONO-SYNCHRONIZED SUBPATHS

| Path  | $\Delta r_{ck}$ | $\Delta r_{tk}$ |
|---|-----------------|-----------------|
| $Lp_1=(p_{13}, t_9, p_{14}, t_{10}, p_{15}, t_{11}, p_{16}, t_{12}, p_{10}, t_6)$ | 8               | $+\infty$       |
| $Lp_2=(p_{13}, t_9, p_9, t_1)$  | 7               | 9               |
| $Lp_3=(p_2, t_2, p_3, t_3)$   | 0               | $+\infty$       |
| $Lp_4=(p_2, t_2, p_8, t_8)$   | 5               | 1               |
| $Lp_5=(p_4, t_4, p_5, t_5, p_{11}, t_1)$  | 9               | $+\infty$       |
| $Lp_6=(p_4, t_4, p_6, t_6)$   | 5               | 3               |
| $Lp_7=(p_{11}, t_7, p_7, t_3)$  | 8               | 5               |
| $Lp_8=(p_{11}, t_7, p_{12}, t_8)$   | 0               | $+\infty$       |

**Step 1.1.1.2:** Computation of

$F(G \setminus Lp' \setminus Lp_3 \setminus Lp_6 \setminus Lp_8 \setminus Lp_2, p_2, 9)$

$\Phi^* = \Phi$  then  $F \Leftarrow 9$

**Step 2:** Computation of  $F(G \setminus Lp' \setminus Lp_4, p_{13}, 1)$

$\Phi^* = \{Lp_1, Lp_2\}$

$$F \Leftarrow \min \begin{cases} \min[1, 18+F(G \setminus Lp' \setminus Lp_4 \setminus Lp_1, p_{11}, +\infty)] = 1 \\ \min[1, 7+F(G \setminus Lp' \setminus Lp_4 \setminus Lp_2 \setminus Lp_8, p_2, +\infty)] = 1 \end{cases}$$

The lower bound of passive robustness is equal to 6 (Margin  $\Leftarrow 6$ )

## Conclusion

The local robustness of a given path in the workshops is analytically built up. Moreover, an algorithm computing a lower value of the maximal time disturbances in manufacturing system with and without assembling tasks is provided.

The use of this lower value improves the monitoring of the industrial processes with time constraints, increasing the productivity and reducing the maintenance costs by improving the availability of the production systems.

Based on two workshops topology, it can be claimed that the computed bound allows checking the death of marks without the generation of many false alarms.

The algorithm has to be studied from a computing complexity point of view in order to figure out if it is really scalable and compatible in the general case.

The computing of the lower value of passive robustness in manufacturing workshops with assembling tasks should be paid attention due to its capability in the elimination of unobservable set of disturbances (Mhalla et al, 2010a), (Mhalla et al, 2010b). Since the passive robustness is based upon variations included in validity time intervals, there is no control loop modification to preserve the required specifications. Indeed, the computing of a lower bound of passive robustness interval allows solving the false alarm problem but not the avoidance of constraint violation.

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